The High Reynolds Number Finite Flat-Plate Cascade

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Abstract

A LAMINAR, incompressible, interacting boundary-layer model is presented for two-dimensional cascades. The interaction between the boundary layer and the inviscid flow is given in terms of a local pressure-displacement relation. Using this model, a previously developed fully implicit/coupled algorithm is shown to increase in efficiency as the cascade spacing is decreased. This behavior is explained in terms of the physical structure of the cascade flowfield and is demonstrated in the finite flat-plate cascade.

Contents

Consider a nonstaggered single-row cascade of symmetric airfoils whose thickness distribution is given by $f(x)$. The Reynolds number Re is referenced to the length of the airfoils and the freestream velocity far upstream of the cascade. In the finite flat-plate cascade $f(x)=0$ and the trailing edges of the plates are at $x=1$. Assuming that the airfoils and the boundary layer are both thin leads to the linear pressure-displacement relation for the inviscid surface speed Ue in terms of the boundary-layer displacement thickness δ,

$$Ue = 1 + \frac{Re^{-1/2}}{h}(Ue\delta)_{x-\infty} + \frac{1}{h}\oint_{-\infty}^{\infty} \frac{d}{dt}[Ue(Re^{-1/2}\delta + f)] \times \coth\left[\frac{\pi(x-t)}{h}\right]dt \tag{1}$$

where x is the streamwise coordinate and h the cascade spacing. This equation can be shown to reduce to the correct single-body relation as $h\to\infty$. In principle, Eq. (1) can be generalized to include cambered airfoils at the angle of attack as well as the staggering of the cascade. However, such a treatment is beyond the scope of this study.

In past interacting boundary-layer solution methods, casting the inviscid flow into such a local pressure-displacement relation has proved very useful. This form of the interacting boundary-layer model eliminates iteration on the full inviscid flowfield and allows the inviscid effects to be directly coupled into the outer edge boundary conditions in the boundary layer, as in the methods of Davis and Werle[1] and Veldman.[2] Following Davis and Werle,[1] Eq. (1) is integrated by parts and element expressions are formed using a midpoint rule, at all times being careful to employ the Cauchy principal value of the integral. For a uniform grid with mesh width Δx, the resulting expression for Ue at the streamwise location x_i is

$$Ue_i = g_i + \frac{1}{\pi\Delta x}\sum_j D(i-j)Ue_j[Re^{-1/2}\delta_j + f_j] \tag{2}$$

where g_i includes effects external to the region being calculated and the one-dimensional array $D(k)$ is given by

$$D(k) = \frac{2\pi\Delta x}{h}\frac{\sinh(\pi\Delta x/h)}{\cos(\pi\Delta x/h) - \cosh(2\pi k\Delta x/h)} \tag{3}$$

As $h\to\infty$, Eq. (3) reduces to the form given by Davis and Werle.[1] The boundary-layer equations and associated boundary conditions are then solved in a completely coupled fashion in a direction normal to the body. Details of this procedure can be found in Refs. 1-3.

In a flow with strong viscous-inviscid interaction, the inviscid flow allows for upstream propagation of information to occur through interaction with the boundary-layer displacement thickness. This means that δ must be relaxed in a global fashion. This is accomplished in the spirit of a line relaxation procedure by repeatedly sweeping the calculation region in an upstream to downstream direction. After some reflection, the convergence rate can be seen to be limited by the amount of upstream influence present in the flow, i.e., by the ratio $|D(k)/D(0)|$ for $k\neq 0$. It can be shown that for the cascade

$$\left|\frac{D(k)}{D(0)}\right|_{h=h_1} < \left|\frac{D(k)}{D(0)}\right|_{h=h_2} \quad \text{for } h_1<h_2 \text{ and } k\neq 0 \tag{4}$$

and

$$\left|\frac{D(k)}{D(0)}\right| \to 0 \quad \text{as } h\to 0 \text{ for } k\neq 0 \tag{5}$$

Equation (4) states that under similar flow conditions, the effect of upstream influence is decreasing as the cascade spacing decreases, implying an increase in the convergence rate. This claim is borne out for the finite flat-plate cascade in Fig. 1.

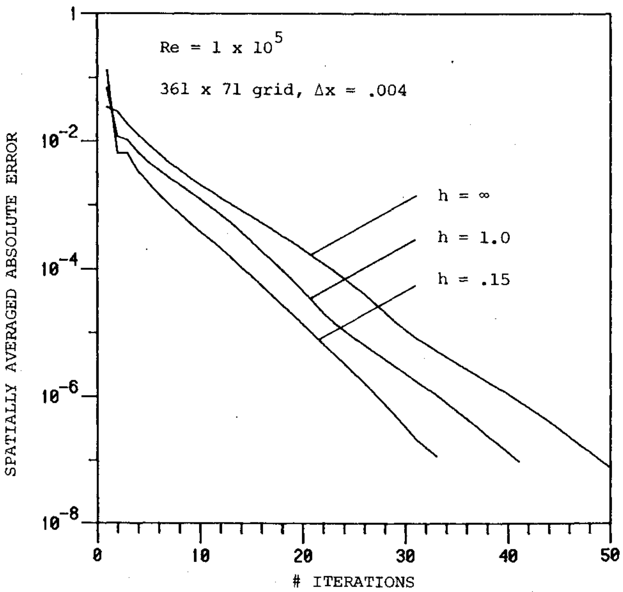

Fig. 1 Convergence properties.

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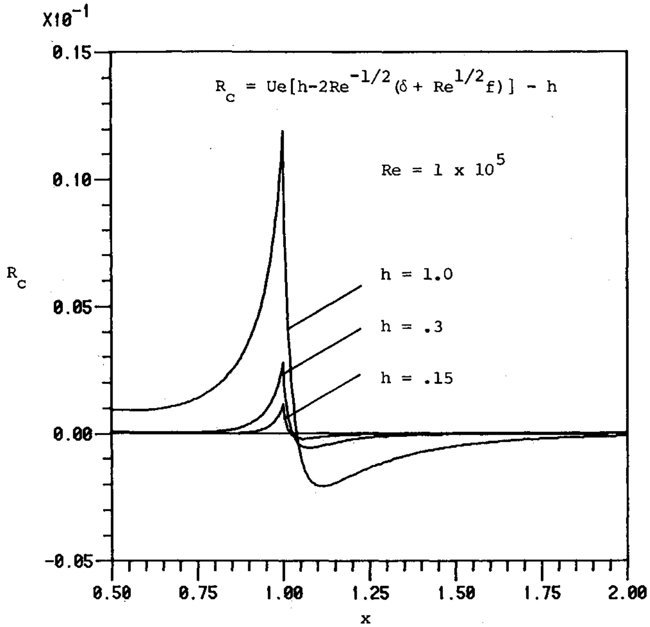

Fig. 2 Deviation from a one-dimensional flow.

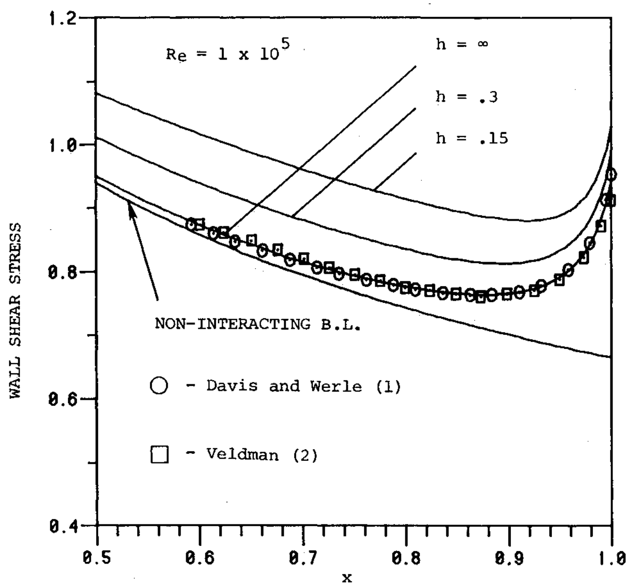

Fig. 3 Wall shear stress.

Equation (5) shows the tendency toward a localized one-dimensional interaction as the cascade spacing is decreased. The pressure-displacement relation [Eq. (1)] can be expanded for $h \to 0$ to give

$$h = Ue[h - 2(Re^{-1/2}\delta + f)] \tag{6}$$

This is simply a one-dimensional mass balance between upstream infinity and a particular x location. As h decreases, the deviation from the one-dimensional flow of Eq. (6) should also decrease. This result is shown for the finite flat-plate cascade in Fig. 2. From Fig. 2, note that a region around the trailing edge is always present in which the full form of Eq. (1) must be used.

At first glance, the result of the increased convergence rate with a decrease in cascade spacing may seem at odds with the intuitive feeling that the interaction effects must be stronger in a cascade due to the confinement of the inviscid flow. In fact, Fig. 3 shows that for the finite flat-plate cascade, the level of interaction is globally increased for decreased cascade spacing. In viewing this figure, one should keep in mind that the $Re \to \infty$ limit solution for all cases is the noninteracting boundary layer.

There are two competing influences in the cascade problem. Even though the level of interaction is increasing, at the same time it is becoming more localized. In addition, the region over which the upstream influence is important actually decreases with decreasing cascade spacing. It is this localization of the interaction process between the viscous and inviscid flow that leads to the increased convergence rate for the cascade problem.

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